



Mathematical Problem Solving: Beliefs, Values, and Implications for Teaching and Teacher Education

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Article history

Received: 11 January, 2026
Accepted: 23 May, 2026
Published: 15 June, 2026

Keywords

Mathematical problem solving, beliefs, values, mathematics education, teacher education

ABSTRACT

Problem solving has become a central objective of mathematics education in many countries, yet instructional practice often continues to equate success with the rapid production of correct, well-presented answers. This perspective argues that an emphasis on correctness is insufficient to account for the nature of mathematical problem solving as an intellectual and cultural practice. Drawing on research in mathematics education, the paper highlights the crucial roles of beliefs and values in shaping learners' engagement with non-routine tasks, their willingness to persist in the face of uncertainty, and their views of what it means to "do mathematics". Beliefs about mathematics, learning, and self-capability interact closely with educational values such as whether speed or depth, outcomes or processes, and competition or collaboration are prioritised in classroom practice. These orientations influence not only how students approach problem solving but also the kinds of learners they become. The article further discusses how teachers and teacher education can support learners in thinking and working more like mathematicians, and illustrates how innovative pedagogical approaches - such as comics-based mathematical tasks - can integrate motivation, values education, and problem solving. It is argued that while correctness remains a necessary condition in mathematics, the cultivation of resilient, reflective, and humane ways of thinking through problem solving should be recognised as a key educational goal.

1. INTRODUCTION

Over the past several decades, problem solving has become a central focus of school mathematics curricula in many countries. Mathematical tasks are no longer designed merely to assess students' ability to apply familiar formulas or procedures; instead, they emphasize situations in which solutions are not immediately apparent (non-routine problems), requiring learners to think, interpret, and experiment. However, in instructional practice, success in problem solving is often equated with producing the correct answer quickly and presenting a neat, standard solution. This approach is insufficient to capture the nature of mathematical problem solving as an intellectual activity and a cultural practice of mathematics. An almost exclusive emphasis on correctness risks obscuring deeper aspects related to beliefs, values, and how people actually 'do mathematics'.

In many problem-solving situations, learners cannot simply apply learned formulas directly (Schoenfeld, 1985, 2016; Toh et al., 2008; 2011). The process requires "getting one's hands dirty," involving applying various rules of thumbs or heuristics to tackle difficult situations, such as trial-and-error, checking one's interpretation of a given problem, solving a simpler problem, and other problem-solving heuristics. This paradigm shift implies that teachers do not always solve problems more quickly or more "elegantly" than students. Teachers could consider thinking

aloud various problem-solving processes and discussing various problem-solving strategies with their students. This is particularly important when dealing with open-ended problems or exploratory tasks.

More importantly, students need to be equipped with a problem-solving model if they are expected to be able to handle non-routine problems like a mathematician independently (e.g., Toh, 2008a). However, mastery of problem-solving models or procedures is necessary but not sufficient to ensure success in problem solving. Such success depends to a large extent on four closely interrelated dimensions: cognitive resources such as knowledge and procedures; problem-solving strategies or rules of thumb; control or metacognition; and, crucially, belief systems (Schoenfeld, 1985).

2. BELIEFS AND VALUES AS FOUNDATIONS OF ENGAGEMENT IN PROBLEM SOLVING

In mathematics education, a belief refers to an idea, an assumption that an individual holds about mathematics, its teaching, and its learning. Citing Schoenfeld, a belief is “an individual’s understanding and feelings that shape the ways that the individual conceptualizes and engages in mathematical behavior” (Schoenfeld, 1992, p. 358).

Belief and value are two closely related but distinct constructs in mathematics education. Belief refers to psychologically held propositions regarded as the truth (Philipp, 2007), while value concerns what individuals consider important, worthwhile, or desirable in mathematical activity (Bishop, 1996). While belief shapes how learners interpret mathematics, value influences what learners prioritize and appreciate within mathematical experiences.

Belief can be understood as individuals’ subjective conceptions of the nature of mathematics, of how mathematics is learned and taught, and of their own capabilities as doers of mathematics. It is common classroom knowledge that teachers’ classroom practices foster related beliefs about mathematics and mathematical problem solving among their students. For example, a teacher who pays much emphasis on speed, rules and correct answers is likely to reinforce the belief that mathematics and mathematical problem solving is about applying memorized facts and rules. As another example, teachers who overly emphasize the application of mathematics in the real world might run the risk of reinforcing the utility value of mathematics at the expense of the aesthetic aspect of mathematics, one dimension that is likely to have attracted mathematicians for centuries (Kissane, 2020). In another case, a mathematics teacher who thinks aloud with their students on how the teacher has struggled through solving selected mathematics problems and eventually managed to get “unstuck” in solving the problems will likely present mathematics differently from one who focuses on presenting to students *the* correct approach to solving problems..

Ernest (1989) discusses collectively three major beliefs about the nature of mathematics: the instrumentalist, Platonist, and problem-solving views. The instrumentalist view perceives mathematics as a collection of fixed rules, formulas, and procedures used to obtain correct answers. The Platonist stance sees mathematics as a coherent and logically connected body of absolute truths that exist independently of human activity. In contrast, the problem-solving view conceptualizes mathematics as a dynamic and creative human endeavor involving exploration, reasoning, and inquiry. Ernest argues that these differing beliefs influence how mathematics is taught and learned, and the values emphasized in mathematics classrooms.

Closely related to beliefs are values - what learners and teachers regard as worthwhile goals to be achieved through mathematics and problem solving. In educational research, values are understood as what individuals or communities consider desirable, worth pursuing, and meaningful in guiding behavior, decisions, and educational goals. Unlike knowledge or skills, values are not what learners “know” or “can do,” but what they consider important and worth striving for in the learning process. According to Clarkson and Bishop (1999), values in education are guiding principles and norms that are formed and reinforced through teaching activities, assessment practices, and social interactions in schools. Values do not exist independently but are closely tied to socio-cultural contexts and the characteristics of specific domains of knowledge. In teaching and learning, values are reflected in what teachers and students prioritize: outcomes or processes, speed or depth, competition or collaboration, adherence to procedures or creativity. Whether explicit or implicit, these values strongly influence instructional organization and learners’ experiences.

As discussed in the preceding paragraphs, different beliefs about the nature of mathematics likely emphasize different values that are underscored in the mathematics lesson. Broadly speaking, we can expect the following correspondence.

Beliefs on the Nature of Mathematics	Views of Mathematics	Associated Values
Instrumentalist	Mathematics as rules/ procedures	Accuracy, Efficiency
Platonist	Mathematics as fixed truths	Certainty, Rigor
Problem-solving	Mathematics as inquiry	Creativity, Reasoning

It should be stressed that we are not making judgement on the relative superiority of each of these three views but attempting to visualize what a mathematics classroom looks like based on the belief and the values emphasized. The lessons of a teacher influenced by instrumentalist view likely focus on imparting sets of rules in solving mathematics problems, with a strong emphasis on accuracy and efficiency of the various mathematical procedures. The lessons of one influenced by the Platonist view likely lay a strong emphasis on mathematical rigor. Advocates of these two views are less likely to engage students in open-ended discussion or tolerate incorrect responses. On the other hand, a problem-solving view of mathematics perceives mathematics as a dynamic inquiry, possibly admitting multiple approaches to solving a single problem, and even allows open discussion of less orthodox approaches of solving mathematics problems.

From this perspective, problem solving is not merely about obtaining the correct solution. It also involves dynamic inquiry: providing learners the opportunity to discover, explore multiple solution approaches to a single problem, extending or generalizing a problem, or even posing a new problem with given parameters. However, it is important to highlight at this juncture that the three beliefs of mathematics are not mutually exclusive. In classroom lessons, a problem-solving based lesson could be one that emphasizes discovery and exploratory learning with opportunities given to students to explore multiple solution approaches and extend the problem. Simultaneously, precision and accuracy of mathematics, and the rigor of mathematics are still reserved due to the disciplinarity of the subject. For example, a solution by trial-and-error to challenging mathematics problems is not acceptable as a rigorous solution in mathematics, although pedagogically trial-and-error is a sound problem-solving heuristic to begin problem solving - elegant and rigorous solution is the end goal of solving a mathematics problem. In the process of experiencing problem solving, students could be invited to experience “doing” mathematics like a mathematician, students could understand that problem solving is time-consuming and requires perseverance, as well as valuing careful thinking, intellectual honesty, and humility – acknowledging what one does not yet understand and being willing to learn from others. These are the universal values of professional mathematicians.

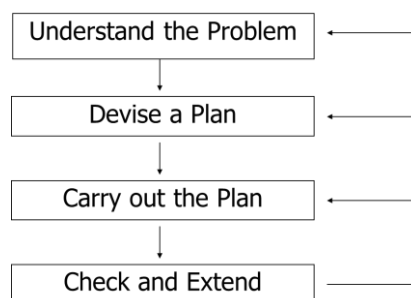
In view of authentic mathematics classrooms, anecdotes show that many students’ beliefs are reinforced by classroom practices shaped by their teachers. Assessments that emphasize completing many tasks within a short time, or teachers’ presentation of only “clean” and polished solutions without sharing their thinking processes aloud, can easily foster the belief that good mathematics students are those who solve problems quickly, make few mistakes, and always present perfect solutions. Such beliefs do not accurately reflect the realities of intellectual work in mathematics, where error, experimentation, and argumentation are integral part of problem solving.

3. SUPPORTING STUDENTS IN PROBLEM SOLVING

As a dynamic inquiry from a problem-solving perspective, mathematics could be presented using approaches which are part of the real world or the world of fantasies instead of presenting in the usual “dry” mathematical problems. Teachers have put in much effort to engage students in diverse mathematical activities which are appealing to students, especially among the unmotivated students and the low readiness learners (e.g., Toh & Lui, 2014). Real-world problems using comics and movie clips were used to engage students in mathematics discourse (e.g., Chu & Toh, 2020; Han & Toh, 2019). In Singapore between 2014 and 2022, two research projects integrated comics to contextualize mathematical problem-solving tasks. This approach aimed to enhance learning motivation, particularly among students with low interest in mathematics, by helping them recognise the relevance of mathematics to everyday life. These two projects emphasize the use of comics to contextualize and teach mathematics, based on the premise that mathematics is meant for the general student population, including those less motivated. At the same time, the tasks were designed to develop 21st-century competencies and convey social values such as environmental sustainability. This case illustrates how mathematics can function as a medium for educating values and beliefs, beyond the sole requirement of solution correctness (Toh et al., 2017). By drawing on familiar contexts and themes,

such as environmental sustainability, mathematics can serve as a vehicle for conveying social values and developing 21st-century competencies, rather than being seen merely as a collection of abstract techniques.

In addition to engaging students with the real-world context and mathematics discourse through less conventional means, it is equally important to engage students in authentic mathematical problem solving. An authentic problem-solving model, e.g., Polya's four-stage problem-solving model, should be introduced to students to enable them to do problem-solving. The issue of problem-solving lesson enactment has been discussed in detail in existing scholarship (e.g., Toh et al., 2008, 2008a, 2011). What needs to be highlighted here is that through lesson enactment, it is crucial to strengthen the belief in students that in addition to being sets of fixed rules or truth, mathematics is also about dynamic human inquiry.



Engaging students in problem-solving allows them to attempt devising their plan, carrying out the plan and switching to another plan if one is at the dead-end. It also means allowing the students to explore, evaluate and compare various solution approaches. This is the practice of meta-cognition in mathematical problem solving – an important dimension of problem solving. Note that the role of teachers in problem-solving should not be reduced to providing students with clear explanations of solution procedures, but to provide scaffolding in order to facilitate students' exploration (e.g., Tay & Toh, 2023).

Furthermore, if mathematics is viewed as a dynamic inquiry, students should also be encouraged to extend problems through adapting, modifying, and generalizing given tasks. However, such classroom practices are unlikely to occur meaningfully unless the teachers believe in the problem-solving nature of mathematics. In other words, the enactment of problem-solving lessons cannot be separated from teachers' beliefs about the nature of mathematics. When teachers view mathematics as a dynamic exploratory human activity, they are more likely to create classroom environments that value inquiry, sense-making, and intellectual risk-taking, rather than merely procedural correctness.

4. CONCLUSION AND IMPLICATIONS

Based on the discussion presented in this paper, teachers' beliefs and values about the nature of mathematics fundamentally shape how mathematics and mathematical problem-solving lessons are enacted in classrooms. The contribution of this paper lies not merely in advocating problem solving as a pedagogical approach, but in unpacking the underlying beliefs associated with problem solving itself. In particular, the paper highlights that problem solving should be understood in relation to a view of mathematics as a dynamic, exploratory, and human inquiry, rather than solely as a body of fixed procedures and correct answers.

Nonetheless, from this perspective, we still argue that the correctness of mathematical solutions remains necessary and important, yet should not be the sole educational goal. Mathematical problem solving also provides opportunities to cultivate broader educational values and dispositions, including resilience, critical thinking, adaptability, collaboration, and humane ways of thinking. Thus, the significance of problem solving extends beyond cognitive achievement toward the formation of learners' beliefs, values, and identities in relation to mathematics.

The discussion also carries important implications for curriculum, assessment, and teacher education. At the policy level, curricula and assessment systems should allow space for mathematical thinking processes, productive struggle, exploration, and open-ended problem-solving tasks. Assessment practices that focus exclusively on final answers risk undermining the deeper educational purposes of mathematical problem solving and may inadvertently reinforce narrow beliefs about mathematics as merely procedural and driven solely by the correctness of answers / solutions.

At the instructional level, greater attention should be devoted to helping both teachers and students reflect on their beliefs about the nature of mathematics. Effective problem-solving instruction depends not only on the enactment of suitable classroom activities, but also is highly dependent on teachers' underlying beliefs about mathematics as inquiry. Teachers who view mathematics as exploratory and dynamic are more likely to create classroom cultures that value questioning, conjecturing, revising, and sense-making. Consequently, teacher education programs should provide opportunities for future teachers to experience authentic mathematical inquiry and to reflect on how mathematicians think, act, and construct knowledge.

One possible avenue is through closer engagement with contemporary mathematicians. For example, at the Singapore National Institute of Education, preservice mathematics teachers in the degree program are assigned academic mentors who are practicing mathematicians. Such mentoring experiences may help preservice teachers not only deepen their mathematical understanding, but also internalize professional values often associated with mathematical practice, such as diligence, intellectual honesty, humility, and passion for inquiry.

In addition, beliefs and values about mathematics may also be nurtured through the history of mathematics. Historical accounts of mathematicians and the development of mathematical ideas (e.g., Burton, 2003) can help students appreciate mathematics as a deep human endeavor shaped by creativity, struggle, and cultural development. Curriculum designers may therefore consider the History of Mathematics as an important component of mathematics education at appropriate levels.

Ultimately, problem solving should not be viewed merely as a teaching strategy, but as a reflection of deeper beliefs about what mathematics is and what mathematics education ought to achieve. Without attending to these underlying beliefs and values, attempts to implement problem-solving pedagogy may remain superficial. A meaningful problem-solving approach therefore requires not only changes in classroom practice, but also a re-examination of how mathematics itself is perceived and valued.

Conflict of Interest: No potential conflict of interest relevant to this article was reported.

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