



## Pre-Service Mathematics Teachers' Beliefs and Instructional Practices Regarding the Teaching of Mathematical Proofs

Tong Huu Duong<sup>1</sup>,  
Nguyen Thai Dao<sup>1</sup>,  
Dung Minh Tang<sup>2</sup>,  
Ngan Kim Lu<sup>1</sup>,  
Khanh Hong Phan<sup>1,3,+</sup>

<sup>1</sup>Can Tho University, Vietnam;  
<sup>2</sup>Ho Chi Minh City University of Education, Vietnam;  
<sup>3</sup>Dong Thap University, Vietnam  
<sup>+</sup>Corresponding author • Email: [phkhanh@dthu.edu.vn](mailto:phkhanh@dthu.edu.vn)

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### ABSTRACT

The beliefs and self-assessed instructional competences of pre-service mathematics teachers regarding the teaching of mathematical proofs play a crucial role in their professional development. This study involves a survey with 295 pre-service mathematics teachers from selected universities in Vietnam's southern region, aiming to explore their beliefs and instructional practices regarding teaching mathematical proofs. Findings indicate that many pre-service teachers felt confident about basic proof concepts and methods, yet a substantial proportion reported insufficient knowledge of mathematical proofs and proof teaching, revealing internal contradictions in their self-perceptions. In addition, their self-assessed teaching competencies do not reflect their beliefs, especially in designing learning activities, organizing instruction, and assessing high school students' performance. The gap between pre-service mathematics teachers' beliefs and instructional practices is a core issue to address in current mathematics teacher training. The results yield important implications for mathematics teacher training and new research directions.

## 1. INTRODUCTION

In recent years, innovation in teaching methods has gained growing attention in Vietnam's education sector, with the aim of improving the quality and effectiveness of educational programs. These innovations require teaching staff with sufficient professional competence to meet the demands of the job and society. Consequently, it is crucial to train proficient pre-service math teachers in order to raise the standard of math education and equip future generations of math teachers with required skills and knowledge. (Anh & Lan, 2019; Hung, 2017).

Mathematics education plays a crucial role in developing students' logical thinking, creativity, and problem-solving skills. Proof is an essential component of the mathematics curriculum, helping students understand the nature of concepts and theorems, as well as developing logical thinking and rigorous reasoning skills. However, teaching and learning proofs effectively still pose multiple challenges for both teachers and students (Haavold et al., 2024; Mingus & Grassl, 1999). Many teachers struggle to convey in-depth math knowledge clearly and vividly, making it boring and difficult for students (Hanna & Knipping, 2020). Therefore, one of the teaching competencies that needs to be developed for pre-service mathematics teachers is the ability to teach proofs (Noto et al., 2019) - an important topic in mathematics education in Vietnam, aiming to help students develop logical thinking, analytical skills, and problem-solving abilities (Er & Dost, 2022; Hanna et al., 2023; Hieu, 2009; Son, 2013).

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Previous research has identified primary causes, including pre-service math teachers' low confidence in their ability to teach proofs (Haavold et al., 2024; Lee, 2026; Mingus & Grassl, 1999) and students' lack of experience and efficient teaching strategies to communicate proof content understandably and interestingly (Mingus & Grassl, 1999). Furthermore, the pre-service math teacher training program has not sufficiently prepared teacher students to instruct proofs (Anh & Lan, 2019; Thang, 2011; Thanh, 2019). Pre-service math teachers must therefore enhance their confidence, acknowledge the value of teaching proofs, and grasp the requisite pedagogical knowledge and skills in order to increase the efficacy of teaching proofs (Haavold et al., 2024; Lee, 2026; Mingus & Grassl, 1999). Besides the fact that teacher training programs provide opportunities for pre-service mathematics teachers to practice teaching proofs and improve their proof-teaching skills (Cuong et al., 2023), the structure of teacher students' beliefs and practices regarding teaching proofs needs to be investigated to provide practical information for educators to adjust mathematics teacher training programs in alignment with practice and actual learners' needs (Capps et al., 2012, cited by Tu & Chi, 2024; Lesseig & Hine, 2021; Mingus & Grassl, 1999). Therefore, conducting a survey to understand pre-service mathematics teachers' beliefs and practices of teaching proofs is necessary (Lesseig & Hine, 2021). However, this research issue has not yet been investigated in the Vietnamese mathematics education context. Thus, this study aims to explore pre-service mathematics teachers' beliefs and self-assessed instructional competences regarding teaching proofs as well as perceived tensions between what they value and what they feel capable of doing in practice within the current mathematics teacher education program. The following questions are addressed to fulfil the research objectives.

1) What are the beliefs of pre-service mathematics teachers regarding teaching proofs, their students' ability to learn proofs, and the role of teachers in teaching proofs?

2) What is the current state of instructional practices among pre-service mathematics teachers regarding teaching proofs, considering three aspects: professional knowledge, design and organization of teaching activities, and assessment and feedback related to teaching proofs?

3) What difficulties do pre-service mathematics teachers face in practicing teaching proofs? How do these difficulties reflect the gap between their beliefs and teaching practices?

## 2. LITERATURE REVIEW

### 2.1. Teaching mathematical proofs

In mathematics education, mathematical proofs are frequently conceptualized as rigorous arguments that adhere to specific community-sanctioned standards of reasoning and validation (Czoher & Weber, 2020; Haavold et al., 2024). Rather than serving solely as tools for verification, the recognized functions of proof have evolved significantly over the past four decades; contemporary curricula emphasize its epistemological roles in explanation, communication, and the discovery of new mathematical knowledge (Hanna & Knipping, 2020). Consequently, integrating comprehensive proof instruction into teacher preparation programs is essential, as it not only deepens pre-service teachers' mathematical comprehension but also equips them with the pedagogical frameworks necessary to foster deductive reasoning in their future classrooms (Haavold et al., 2024; Stylianides et al., 2023).

As it fosters students' appreciation, comprehension, and mastery of mathematical concepts, teaching proofs in mathematics is critical (Er & Dost, 2022; Hanna & Knipping, 2020). In addition to demonstrating the accuracy of claims in the creation of new knowledge, proofs improve problem-solving abilities and deepen comprehension of mathematics (Clarke, 2023). However, teaching proofs presents difficulties, such as balancing different pedagogical approaches in mathematics education and requiring rigorous reasoning to establish relationships among mathematical elements (Hanna & Knipping, 2020). All things considered, teaching proofs is essential to students' growth in mathematical competence, problem-solving skills, and mathematical reasoning.

### 2.2. Beliefs of pre-service mathematics teachers

It has been demonstrated that teachers' beliefs significantly impact how they decide to teach and how they use technology and pedagogical approaches in the classroom (Havelková et al., 2022; Kul, 2018). Research in mathematics education frequently distinguishes between beliefs about the nature of mathematics and beliefs about teaching and learning mathematics (Lee, 2019). We use this framework in this study to investigate pre-service teachers' beliefs regarding mathematical proof and proof instruction. In mathematics education, teachers' beliefs are considered in two aspects: beliefs about the nature of mathematics and beliefs about teaching practices (Kul, 2018).

In this study, the beliefs of teacher trainees regarding evidence-based teaching are also examined in the same two aspects: beliefs about proof and beliefs about teaching proof (Lee, 2026). Specifically, beliefs about proof include beliefs about the meaning and role of proof in mathematics education, while beliefs about teaching proof include beliefs about teaching ideas, students' abilities, and teachers' roles in teaching and learning proof (Frasier & Panasuk, 2013; Kotelawala, 2016; Lesseig & Hine, 2022). It has been found that teachers believe using proof enhances their students' mathematical abilities, including their ability to reason logically, argue, and solve problems (Mingus & Grassl, 1999). From the student perspective, some studies indicate that teachers believe that proofs are not easily accessible to all students and are more suitable for advanced students with a need for learning (Lee, 2019) due to their high complexity and abstraction, and because some students lack interest or intention in learning proofs (Lesseig et al., 2019). Meanwhile, many teachers believe that with the teacher's role in guiding and facilitating students' knowledge formation, participation in discussions and reasoning in proofs, and with appropriate teaching and learning methods, the majority of students can still access mathematical proofs (Furinghetti & Morselli, 2011).

### **2.3. Instructional practice of pre-service mathematics teachers**

Numerous studies have shown that teachers' beliefs influence their teaching methods, which, in turn, affect their students' learning outcomes (Tassell et al., 2019). A key component of teacher preparation programs is instructional practice. Gower et al. (2009) define instructional practice as the process by which prospective math teachers use their teaching techniques and abilities in an actual classroom setting, assisting them in acquiring the professional and personal skills required for the field (Kasanda, 1995). Therefore, instructional practice plays a very important role in the career development of pre-service mathematics teachers (Azhar & Kayani, 2017). In proof-based teaching, the teaching practices of mathematics teachers are considered in areas such as understanding proof-based teaching methods, organizing proof-based instruction, and student assessment (Mingus & Grassl, 1999). In this context, teachers' beliefs about their role in evidence-based teaching are crucial to how it is organized, with two approaches: content-focused and learner-focused, corresponding to two perspectives on the teacher's role: transmitting knowledge or guiding students in knowledge formation (Lee, 2026).

## **3. MATERIALS AND METHODS**

This study adopts a quantitative descriptive survey design. Data were collected through a self-administered questionnaire and analyzed using descriptive statistics. The participants are 295 pre-service mathematics teachers enrolled in undergraduate teacher education programs at five universities in Southern Vietnam. The sample included both male and female students, with the majority in their third or fourth year of study, and was obtained through convenience sampling. Although the sample size is adequate for descriptive purposes, its institutional distribution is skewed towards two universities; therefore, the findings should be interpreted as indicative rather than statistically representative of all pre-service mathematics teachers in the region.

The questionnaire was developed by the research team based on previous studies on teachers' beliefs and proof teaching and consisted of four main parts: (1) background information; (2) beliefs about the role and significance of mathematical proof in school mathematics, students' ability to learn proofs, and the teacher's role in proof instruction; (3) self-assessed instructional competences related to teaching mathematical proofs and (4) perceived difficulties in teaching proofs. Parts (2) and (3) used 5-point Likert-scale items, whereas Part (4) comprised multiple-choice items allowing participants to select more than one difficulty. The initial item pool was drafted in Vietnamese and then reviewed by several mathematics education experts to ensure content relevance, clarity, and alignment with the Vietnamese secondary mathematics curriculum. In response to their comments, unclear language was revised, overlapping items were eliminated, and examples were modified to better reflect pre-service teachers' experiences. The procedure helped establish the instrument's content validity. In this study, instructional practices are operationalized as self-assessed instructional competences reported on the questionnaire, rather than as directly observed classroom behaviors.

The online survey was carried out using Google Forms. The survey link was distributed via class groups and institutional mailing lists with the help of instructors at the participating universities. Participation was anonymous and voluntary. No identifying personal data was gathered, including names or student ID numbers. After the data was exported, it was screened and analyzed using Microsoft Excel. To characterize patterns in pre-service teachers' beliefs, self-assessed instructional competencies, and perceived challenges in teaching mathematical proofs, frequencies, percentages, means, and standard deviations were computed. Since the purpose of the study was to give

an exploratory, descriptive account of the current situation, no inferential statistical tests were performed. The discussion claims the drawbacks of depending only on self-reported data and descriptive statistics.

## 4. RESULTS AND DISCUSSIONS

### 4.1. *Demographics of the pre-service mathematics teachers participants*

Participant demographic information included gender, current academic year, and the university of the participating pre-service mathematics teachers. Regarding gender, male pre-service mathematics teachers accounted for 52.9% and female pre-service mathematics teachers 47.1%. The relatively balanced male-female ratio, with insignificant gender differences, contributes to the generalizability and representativeness of conclusions about pre-service mathematics teachers' beliefs and practices. Regarding the current academic year, third and fourth-year pre-service mathematics teachers constituted the majority (80%). This group has participated in, or is currently participating in, teaching methodology and instructional practice courses, and their assessments are highly reliable for the study of demonstration teaching practices.

Regarding educational institutions, the data shows participation from pre-service mathematics teachers from many major universities in the Southern region. Among them, Dong Thap University (147 pre-service mathematics teachers) and Ho Chi Minh City University of Education (103 pre-service mathematics teachers) were the two main sources of data. The presence of pre-service mathematics teachers from Can Tho University (23), An Giang University (18), and Saigon University (4) helps broaden the scope of the research and reflects the diversity of teacher education programs in the region. However, because the institutional distribution of the sample is skewed, the findings should be interpreted as indicative rather than fully representative of mathematics teacher training in Southern Vietnam. In short, the survey sample has high statistical reliability thanks to gender balance and, especially, its focus on graduating pre-service mathematics teachers, who are directly transitioning from "belief" to "practice" in their profession.

According to the survey's findings, 75% of the pre-service math teachers participants have taken a course on "teaching proof" in mathematics. This figure demonstrates how teacher preparation programs have appropriately identified and incorporated mathematical proof as a fundamental aspect of pedagogical knowledge. It can be said that the majority of participants have a theoretical basis about the methods, logical structures, and approaches to creating proofs. However, the remaining 25% suggest that student learning progress varies or that training programs are inconsistent across institutions.

Only 39% of the respondents had the chance to practice teaching proofs in instructional practice courses or during teaching internships, which stands in sharp contrast to the theoretical claims. Although 75% had learned the theory, only 39% had the opportunity to apply it in actual teaching. This reflects the lack of practical opportunities in training programs, and the teaching demonstration remains primarily theoretical, neglecting the development of practical professional skills. Furthermore, the low rate of practical training may stem from the fact that during teaching internships, pre-service mathematics teachers tend to choose or be assigned lessons on calculations and formulas (algebra, calculus), which are easier to implement than lessons requiring purely deductive reasoning. Pre-service math teachers are receiving more theory than hands-on opportunities to hone their practical professional skills, as evidenced by the discrepancy between the learning rate (75%) and the practical training rate (39%). This demonstrates unequivocally the necessity of developing strategies to improve hands-on training, simulations, and demonstration lessons with observable content.

### 4.2. *The beliefs of pre-service mathematics teachers regarding teaching mathematical proofs*

#### 4.2.1. *Beliefs about the role and significance of proofs in mathematics teaching*

Exploring beliefs about the roles and importance of proof in mathematics instruction is especially crucial in research on pre-service math teachers' beliefs and teaching practices, as these beliefs influence future teachers' attitudes, motivations, and lesson-planning strategies. The survey results on this issue are presented in Figure 1.

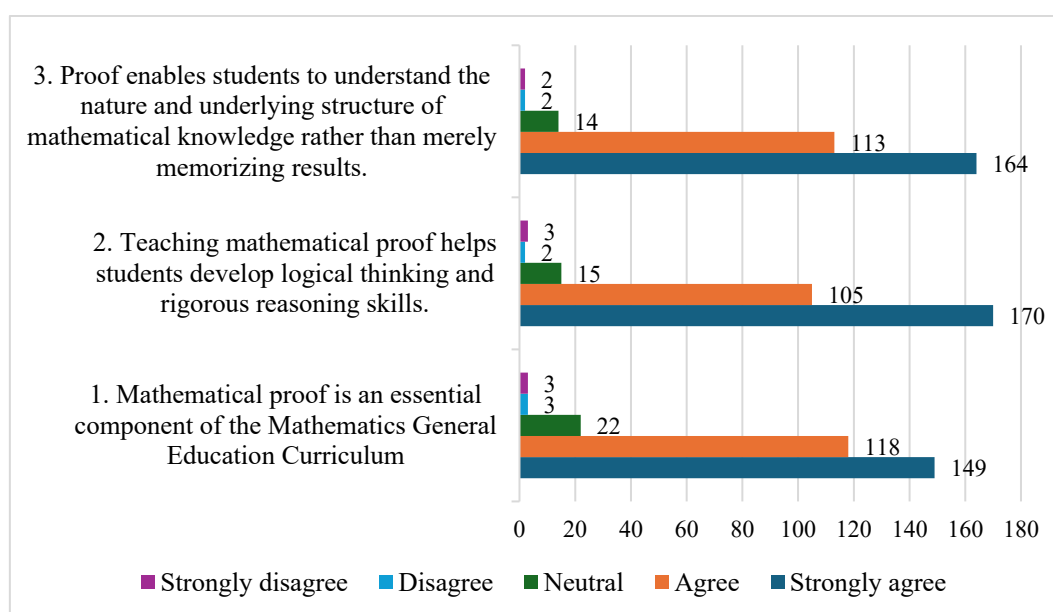


Figure 1. Statistical results of beliefs about the role and significance of proofs in mathematics teaching

The survey results show that the pre-service mathematics teachers had a very positive, fairly unified belief in the role and significance of proofs in teaching mathematics. First, regarding the statement "Mathematical proof is an essential component of the Mathematics General Education Curriculum," 149 pre-service mathematics teachers strongly agreed, accounting for over 50% of the total survey participants. The majority of pre-service math teachers clearly understood the crucial role that proofs play in helping students develop their mathematical thinking, as evidenced by the extremely low percentage who disagreed. Then, with 170 pre-service math teachers strongly agreeing and 105 agreeing, the statement "Teaching mathematical proof helps students develop logical thinking and rigorous reasoning skills" received the highest level of agreement. The 2018 General Education Mathematics Curriculum includes reasoning and mathematical thinking skills as one of the core competencies of mathematics, and this finding reflects pre-service math teachers' understanding of proof-based learning as a tool for developing critical thinking.

164 pre-service math teachers agreed with the statement, "Proof enables students to understand the nature and underlying structure of mathematical knowledge rather than merely memorizing results," and 113 strongly agreed. In line with contemporary teaching philosophies that prioritize in-depth comprehension and application of knowledge over rote memorization, this demonstrates that the surveyed pre-service math teachers had an adequate understanding of the cognitive and pedagogical value of proofs. A small percentage, however, selected "neutral" or "disagree," suggesting that their understanding of the profound significance of proofs remains incomplete. Overall, the results confirm that the pre-service mathematics teachers had a positive belief about the role of proofs in mathematics teaching, creating a favorable premise for developing their competence in teaching proofs in subsequent training and professional development.

#### 4.2.2. Beliefs about students' ability to learn mathematical proofs

Besides assessing pre-service mathematics teachers' beliefs about the role and significance of proofs in mathematics teaching, the study also surveys their views on high school students' ability to learn proofs. This is considered an important factor directly influencing pedagogical expectations and the way pre-service mathematics teachers organize teaching activities when they become teachers. The results are shown in Figure 2 below.

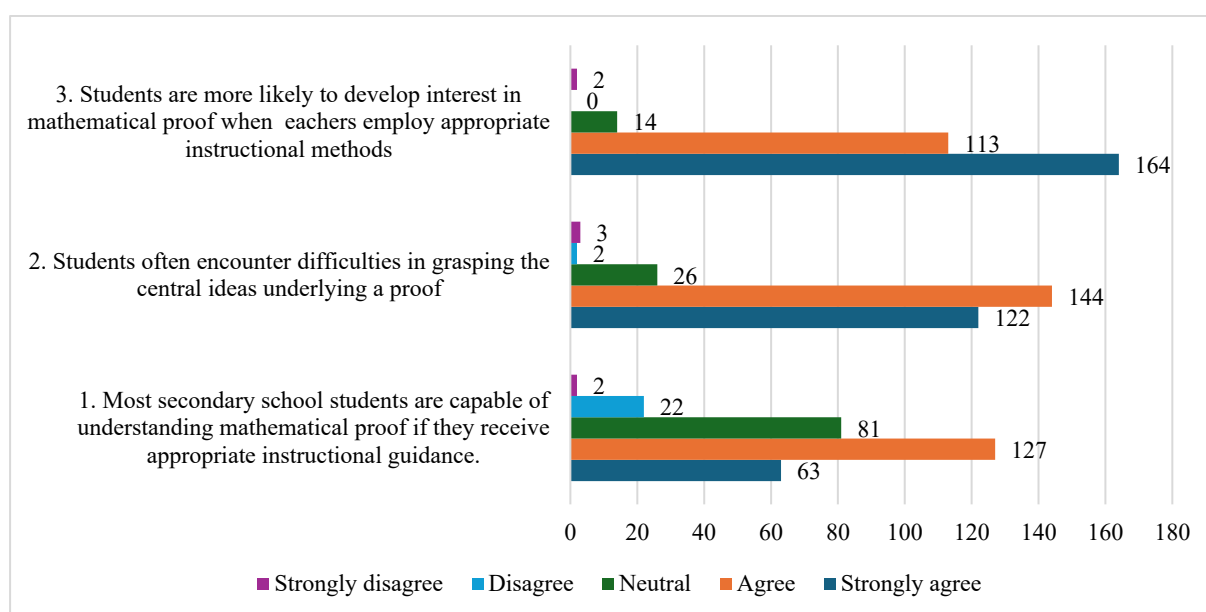


Figure 2. Statistical results of pre-service mathematics teachers' beliefs about high school students' ability to learn through proof-based learning

The survey results show that these pre-service mathematics teachers had a fairly realistic but also relatively positive view of high school students' ability to learn proofs. 64% of pre-service math teachers agreed and strongly agreed with the statement, "Most high school students are capable of understanding mathematical proof if they receive appropriate instruction," while 22 pre-service math teachers (roughly 7.5%) were merely somewhat hesitant. Furthermore, a small proportion (roughly 6%) lacked confidence in students' abilities, indicating a lack of understanding of the real-world constraints in high school proof content. Notably, more than 90% of pre-service math teachers admitted that students frequently struggle to grasp the main idea of proofs, demonstrating a thorough understanding of the difficulties students may face and the complexity of mathematical reasoning. This demonstrates their awareness of the psychology and learning traits of high school pupils.

Additionally, almost 94% agreed with the statement "Students are more likely to develop interest in mathematical proof when teachers employ appropriate instructional methods". This result shows that the pre-service mathematics teachers placed their trust in the decisive role of teaching methods in promoting high school students' interest and ability to participate in practical activities. Thus, despite being aware of the existing difficulties, they still believed that with proper teaching methods, high school students can fully access and perform mathematical proofs. Overall, the data indicate that these pre-service mathematics teachers' beliefs are positive but not idealistic, accurately assessing students' potential abilities while clearly identifying the challenges in teaching proofs. This is a favorable sign for them to develop a sound pedagogical perspective and to choose appropriate teaching strategies in future teaching practice.

#### 4.2.3. Pre-service mathematics teachers' beliefs about the role of teachers in teaching and proving

After exploring pre-service mathematics teachers' beliefs about the role of proof in mathematics teaching and their views on high school students' ability to learn proofs, the study further examines their beliefs about teachers' roles in teaching proof. This is a crucial aspect, as beliefs about professional competence and responsibility will directly influence how they develop their teaching styles in the future. The survey results are shown in Figure 3 below.

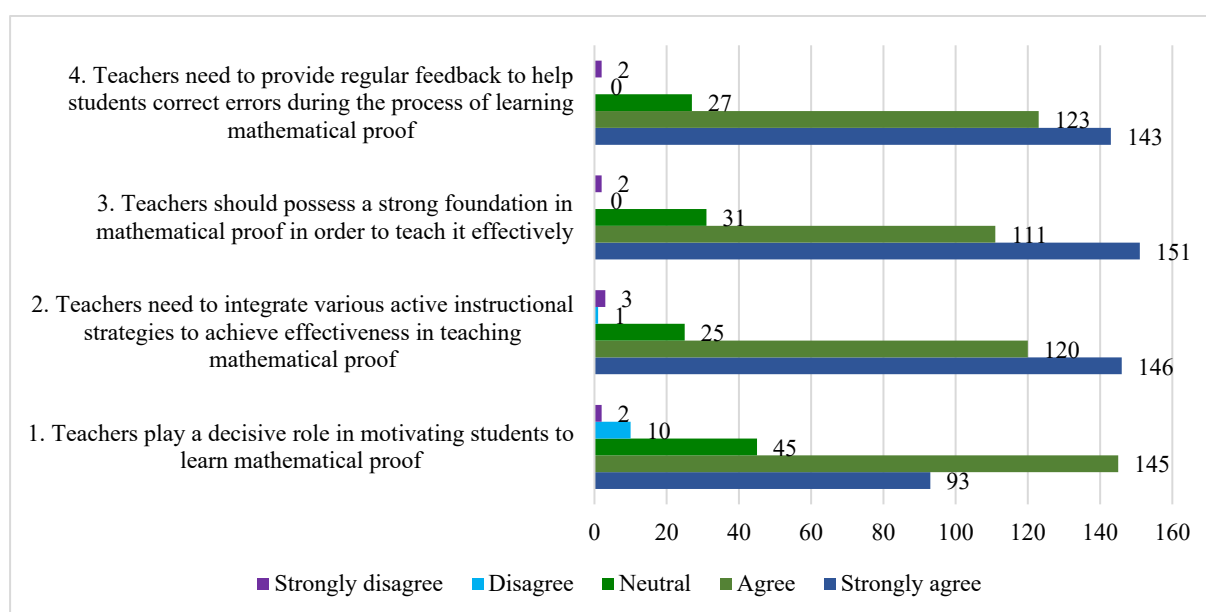


Figure 3. Statistical results of the beliefs of pre-service mathematics teachers about teachers' role in teaching proofs

As shown in Figure 3, the pre-service math teachers had a favorable opinion of the teacher's role in planning the proofs teaching. In particular, more than 64% of the respondents agreed or strongly agreed with the statement "Teachers play a decisive role in motivating students to learn mathematical proof," indicating that most pre-service math teachers surveyed posited that students' learning motivation, particularly with challenging content like proofs, depends largely on the teacher's methods, strategies, and support. This is consistent with the 2018 General Education Mathematics Curriculum's competency-based development orientation, which emphasizes teachers' leadership and guiding roles in students' learning activities. Additionally, more than 90% claimed that students can be interested in proofs when the teacher uses the appropriate techniques, further supporting their belief in the teacher's proactive role. This percentage shows how well they understand the value of methodological innovation, particularly the improvement of active, exploratory, and question-and-answer approaches in teaching proofs. Nearly 94% concurred that students frequently face challenges when learning proofs and require appropriate pedagogical intervention, which supports their belief that teachers must be able to assist their students in overcoming difficulties in understanding the main ideas of proofs. Nonetheless, a minority of the participants continued to voice doubts about teachers' ability to make decisions, as indicated by those who selected "neutral" or "disagree." This could result from a view that still supports total student autonomy in learning or from a lack of practical experience. Overall, however, the findings demonstrate that pre-service math teachers are well aware of the teacher's role in the process of learning proofs as an organizer, supporter, facilitator, and motivator in addition to being a knowledge transmitter.

These findings are consistent with previous international studies reporting that pre-service and in-service mathematics teachers generally hold positive beliefs about the value of proof for developing students' mathematical reasoning and deep understanding (Mingus & Grassl, 1999; Hanna & Knipping, 2020; Lesseig & Hine, 2022). However, the Vietnamese context is characterized by high-stakes examinations and limited instructional time for exploratory activities, which may make it difficult for teachers to realize these beliefs in everyday classroom practice. The strong endorsement of the importance of proof among our participants, therefore, represents a valuable resource that teacher education programs can build on, provided they address the practical constraints of proof teaching in high schools.

#### 4.3. Pre-service mathematics teachers' instructional practice of teaching proofs

##### 4.3.1. General understanding of mathematical proofs and teaching mathematical proofs

Continuing the analysis of pedagogical beliefs, this study focuses on evaluating pre-service mathematics teachers' teaching practices of teaching proofs. This section particularly discusses their professional and pedagogical understanding of mathematical proofs – a crucial foundation for developing competence in teaching proofs in practice. Figure 4 displays the survey results.

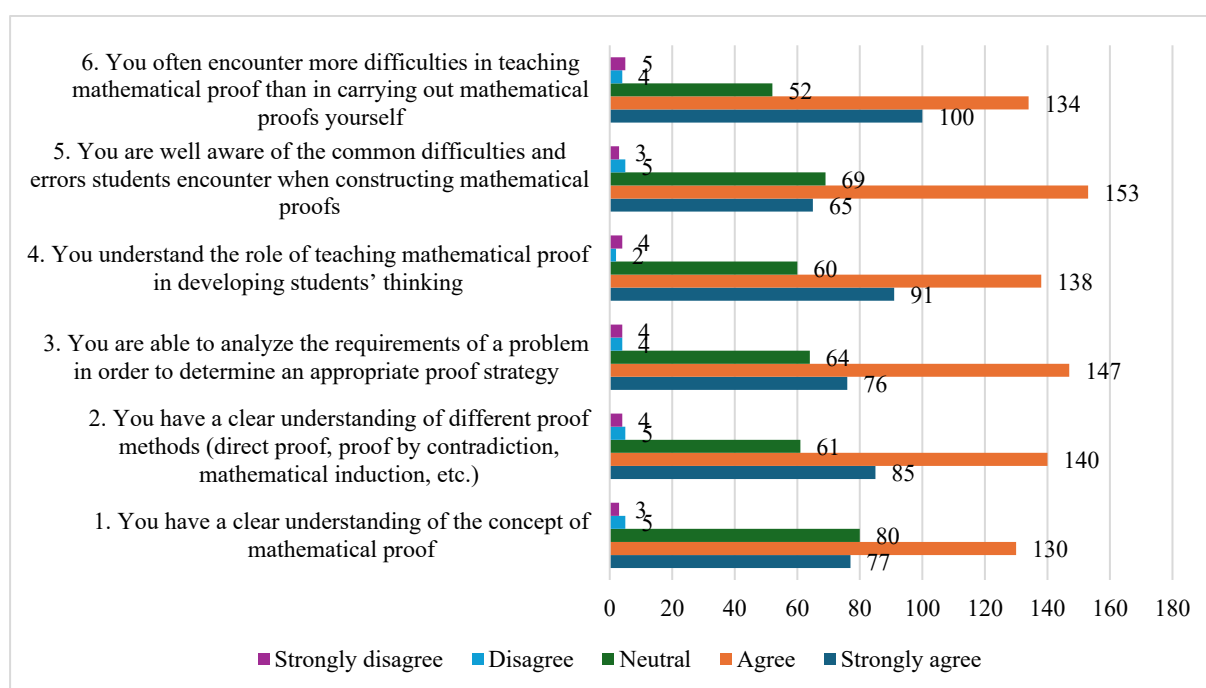


Figure 4. Statistical results of general understanding of mathematical proofs and teaching mathematical proofs among pre-service mathematics teachers

The survey results show that the pre-service mathematics teachers had a fairly positive assessment of their understanding of mathematical proofs and teaching them. Regarding statements related to professional knowledge, the percentages of respondents who agreed and strongly agreed are high. Specifically, 77 pre-service mathematics teachers strongly agreed that they clearly understood the concept of mathematical proofs, and 225 pre-service mathematics teachers were confident in their understanding of proof methods (direct, by contradiction, induction, etc.). This reflects a fairly solid foundation in mathematics among the teacher students. In addition, they also demonstrated an adequate understanding of pedagogical aspects. More than 75% (130 pre-service mathematics teachers agreeing and 77 strongly agreeing) believed they could analyze the requirements of a problem to determine the appropriate proof approach, and 229 pre-service mathematics teachers (77.6%) believed they clearly understood the role of teaching proofs in developing students' thinking skills. Specifically, 218 pre-service mathematics teachers (73.9%) affirmed that they understood the common mistakes students make when performing proofs, a core element that helps teachers provide appropriate support in teaching. However, it's noteworthy that 234 pre-service mathematics teachers agreed, and 52 neutral that they often had more difficulty teaching proofs than performing mathematical proofs themselves. This reflects a clear gap between specialized knowledge and its application in practical teaching, consistent with the current state of teacher training: pre-service mathematics teachers may be able to prove theorems well but struggle to guide their students in proving them. Thus, although pre-service mathematics teachers have a relatively good theoretical understanding, they still lack confidence when implementing classroom teaching activities. This emphasizes the importance of enhancing practical experience, teaching practice, and pedagogical simulations of teaching proofs during the training process.

#### 4.3.2. Pre-service mathematics teachers' practical teaching skills of organizing and implementing demonstration teaching activities

After surveying pre-service mathematics teachers' theoretical understanding of mathematical proofs and of teaching proofs, the study continues to assess their practical teaching competencies. This is a crucial part of future educators' professional competency, as it demonstrates how they can apply pedagogical knowledge to plan and deliver instructional activities. The survey results on pre-service math teachers' practical abilities in carrying out mathematical proof-related teaching activities are shown in Figure 5 below.

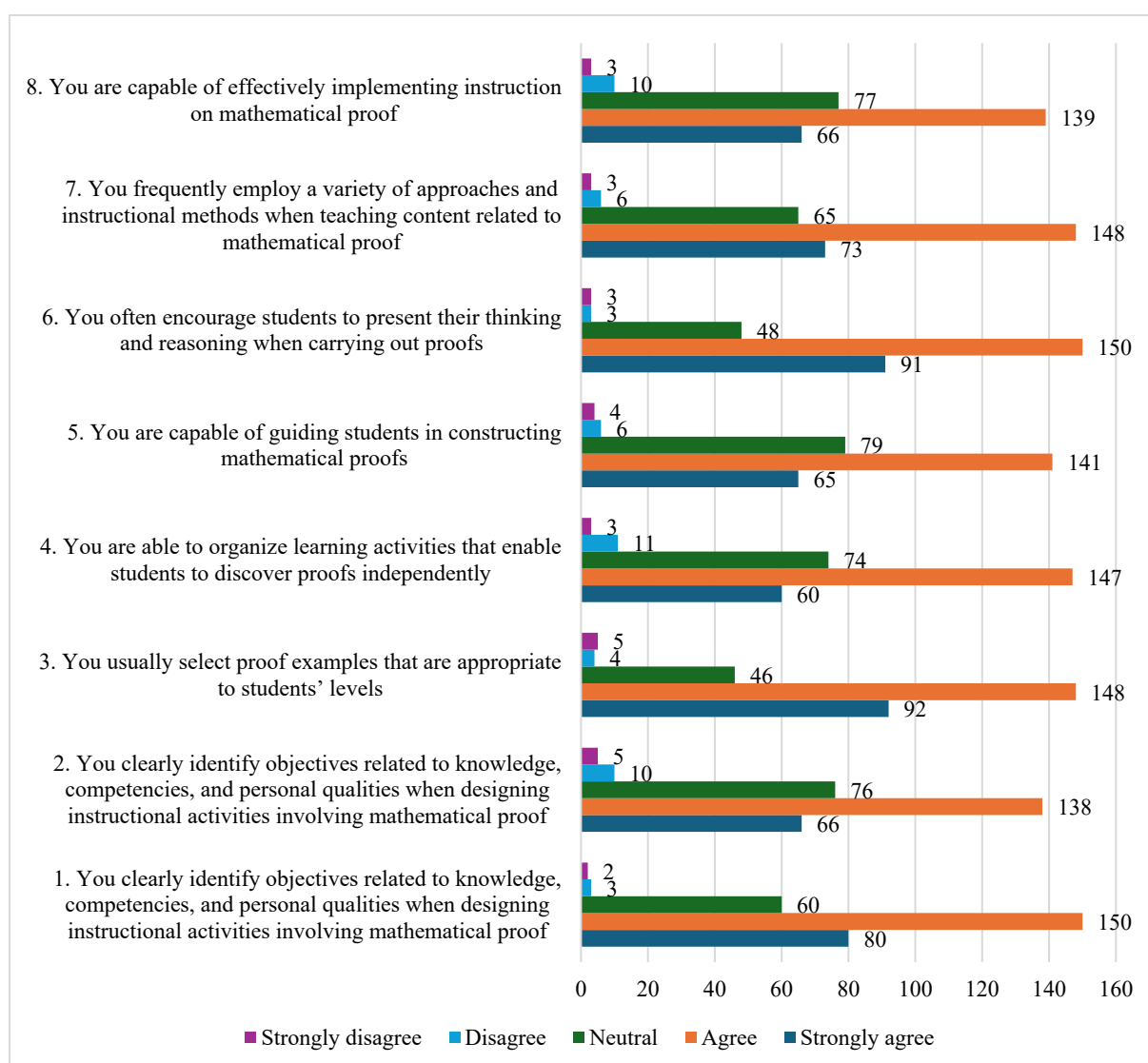


Figure 5. Statistical results of pre-service mathematics teachers' instructional practices and skills of organizing and implementing teaching activities

The survey results show that the pre-service mathematics teachers generally had a fairly positive assessment of their practical abilities in teaching proofs; however, some notable limitations remain. First, according to the 2018 General Education Curriculum, 230 out of 295 pre-service math teachers (78%) confirmed that they can clearly define teaching objectives related to proof-based learning in the group of skills related to defining lesson objectives, a critical foundation of pedagogical competence. This indicates a solid understanding of competency-based development. However, regarding the skill of designing instructional activities involving mathematical proof, 204 pre-service teachers (69.2%) agreed or strongly agreed, 76 (25.8%) adopted a neutral stance, and 15 (5%) disagreed or strongly disagreed with the skill of creating lessons or activities that include evidence-based content, suggesting that this remains a major challenge. This illustrates the disconnection between theoretical knowledge and its application in real-world lesson planning. Subsequently, 92 pre-service math teachers strongly agreed, and 148 agreed that they had a good grasp of how to differentiate instruction to choose and apply teaching content. On the other hand, 74 pre-service math teachers rated their ability to plan learning activities that allow them to learn and prove things for themselves as only average, indicating limited use of active teaching techniques in the classroom.

Among the skills related to supporting and coordinating student learning activities, 206 pre-service mathematics teachers (approximately 70%) expressed confidence in their ability to guide students in proving proofs. Similarly,

the majority stated that they often encouraged their students to present their thoughts and arguments when proving proofs (over 75% high agreement). However, a significant number of respondents, 79 teacher students, chose a neutral rating for the statement about their ability to guide their students in proving proofs, and 48 pre-service mathematics teachers chose a neutral rating for the question about whether they often encourage their students to present their thoughts and arguments when proving proofs. This indicates that a large proportion of respondents are still not truly familiar with supporting, guiding, and encouraging their students in proving proofs. Finally, regarding the skills of using diverse approaches and teaching methods, and the ability to practice teaching proof content, the results show that although the majority claimed these skills (75% and 70%, respectively), some still admitted limitations in practicing teaching proof content.

Overall, the survey's findings show that the pre-service math teachers have acquired essential skills for teaching proof-based math, particularly in content selection, goal-setting, and guiding students' thinking. Nonetheless, highly practical abilities such as planning instructional activities, fostering student self-discovery, and coordinating classroom reasoning remained common constraints. This suggests that, to help teachers better grasp the practical demands of teaching proof-based mathematics in high schools, teacher training programs should include additional pedagogical practice activities, such as teaching demonstrations, model lessons, and professional development sessions centered on lesson study.

#### 4.3.3. Pre-service mathematics teachers' ability to test and evaluate students' learning of mathematical proofs

One of the most important aspects of a math teacher's competency in teaching proofs is their ability to assess students' understanding of proofs, as well as to plan and coordinate instructional activities. This illustrates the extent to which pre-service math teachers can track their students' reasoning development, spot mistakes, and offer pertinent pedagogical feedback to foster the development of proof-based abilities. The chart below shows survey results of pre-service math teachers' testing and evaluating skills.

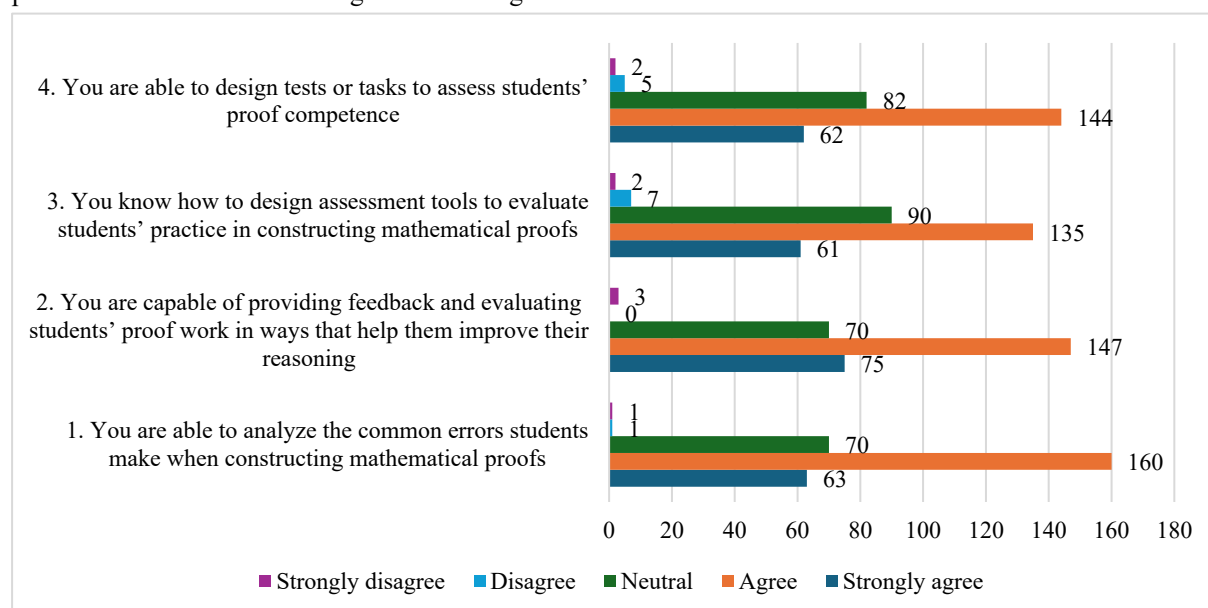


Figure 6. Statistical results of pre-service mathematics teachers' ability to test and evaluate students' learning of mathematical proofs

According to Figure 6, the survey results indicate that the pre-service mathematics teachers had a relatively positive perception of their ability to test and evaluate students' learning of mathematical proofs. However, their confidence is not yet very high, and certain limitations remain. First, regarding analyzing common student errors in proofs, the statement received the highest level of agreement, with 160 pre-service mathematics teachers agreeing and 63 strongly agreeing. This implies that most pre-service math teachers have mastered the ability to spot mistakes in mathematical reasoning, a crucial skill for helping students fix errors and develop their proof-based thinking. Only 70 students, however, chose an average rating, indicating a lack of confidence in the synthesization of common student error types. These teacher students were well aware of the importance of feedback in enhancing the quality

of mathematical reasoning, as evidenced by the 147 pre-service math teachers who agreed and 75 strongly agreed that they can evaluate students' proofs and offer feedback. Nonetheless, the fact that a sizable percentage of 70 pre-service math teachers still opted for an average rating suggests they might find it challenging to offer insightful feedback rather than merely declaring it correct or incorrect.

The results indicated a much lower level of student confidence in the two aspects directly related to the design of assessment tools. Notably, 90 pre-service math teachers selected the "average" level, and only 191 agreed that they knew how to create assessment tools for real-world, evidence-based activities. Similarly, although 202 pre-service math teachers agreed they could create tests or exercises to evaluate proof-based skills, a sizable percentage did not feel fully confident (82 at the "average" level and 7 disagreed). This implies that designing tests of students' proof-based skills remains challenging and calls for a blend of pedagogical thinking and understanding of mathematical content. Overall, the findings demonstrate that the pre-service math teachers were somewhat adept at identifying mistakes and offering feedback on students' proof-based work, but they were still unable to develop assessment instruments and forms that meet the standards for developing competencies. This necessitates enhanced training for pre-service mathematics teachers in process assessment, rubric assessment, analysis of proof-based assignments, and competency-based task design for assessment, to comprehensively improve their competence in teaching mathematical proofs.

#### 4.3.4. Difficulties faced by pre-service mathematics teachers in practicing teaching mathematical proofs

Besides surveying beliefs and self-assessed instructional practice competence, the study also aims to analyze the difficulties that pre-service mathematics teachers encounter in teaching mathematical proofs. Identifying these difficulties is important, as it helps clarify the causes hindering the formation and development of teaching competence in mathematical proofs, thereby providing a basis for proposing appropriate training measures. The survey results are presented in Figure 7.

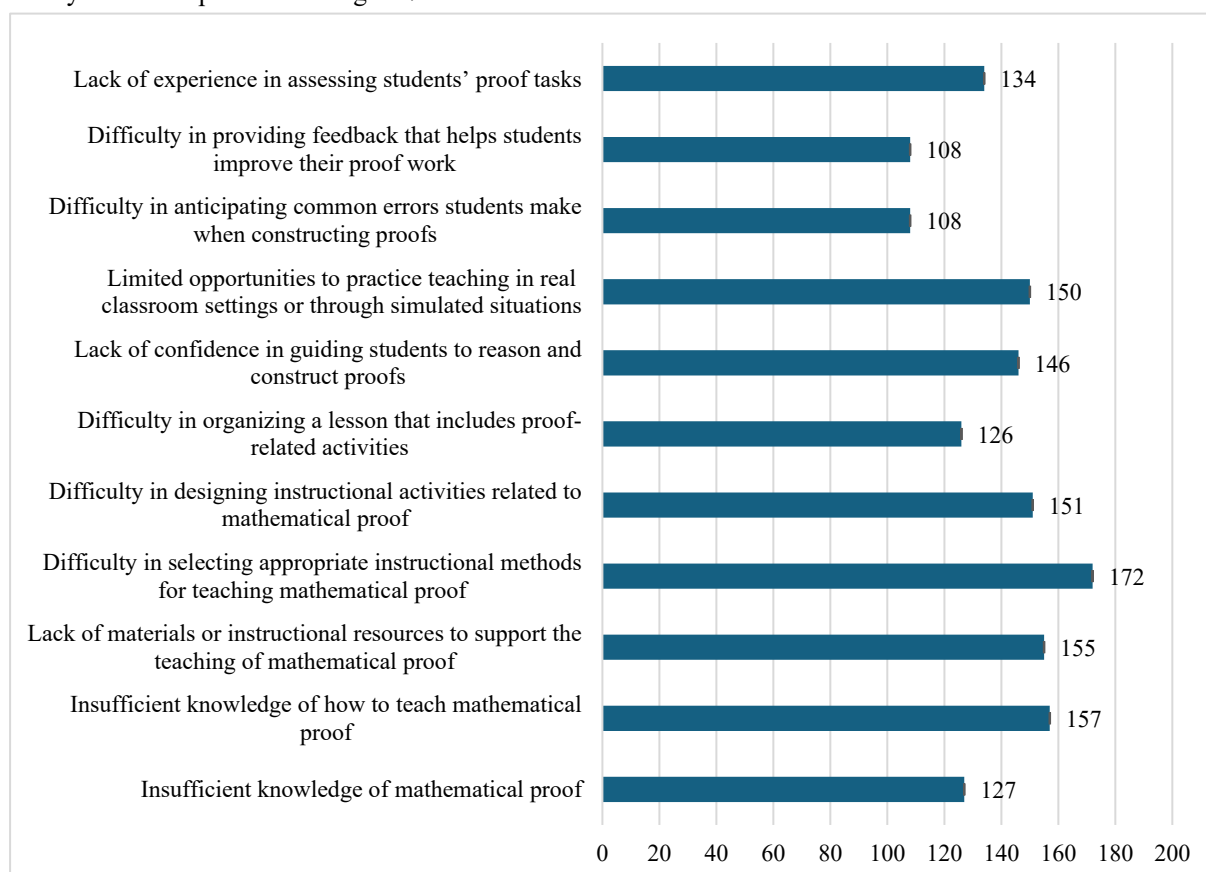


Figure 7. Statistical results of difficulties faced by pre-service mathematics teachers in practicing teaching mathematical proofs

The survey results show that the pre-service mathematics teachers faced many systemic difficulties in teaching mathematical proofs, most notably in teaching methods, activity design, practical experience, and student assessment. First and foremost, 172 responses, choosing a suitable approach for teaching proofs was the most often selected challenge, which illustrates that teaching proofs is a challenging task that requires teachers to be adaptable in their use of techniques such as questioning, group discussion, prompting, and discovery. These pre-service math teachers, on the other hand, had insufficient practice and experience, which prevented them from selecting appropriate approaches given the material and their level. Furthermore, a large number of pre-service math teachers claimed challenges in designing instructional activities. There were 151 votes for challenges in designing proof-based activities and 126 votes for challenges in planning a lesson that included mathematical proof-related activities. The figures demonstrate that the pre-service math teachers have not yet fully grasped how to design and deliver a proof-based lesson using a competency-based approach, particularly in designing a sequence of logical learning exercises that encourage students' critical thinking and reasoning.

Notably, these pre-service mathematics teachers faced significant difficulties regarding practical experience, as evidenced by 150 votes for the lack of opportunities for practical teaching experience in the classroom or through simulated scenarios. The lack of practical experience leads to a lack of confidence in guiding their students in reasoning and proof execution (146 votes), as well as difficulties in predicting common student errors (108 votes) and providing appropriate feedback (108 votes). Furthermore, difficulties related to resources and background knowledge also accounted for a significant proportion, such as a lack of materials or learning resources to support teaching proofs (155 votes), a lack of knowledge about teaching mathematical proofs (157 votes), and a lack of knowledge about mathematical proofs themselves (127 votes). It can be indicated that the current training program needs to further the provision of learning materials, in-depth guidance, and specific illustrative models for teaching proofs.

The pattern of reported difficulties – such as lack of knowledge about proof teaching, limited experience in anticipating students' errors, and challenges in designing proof-related tasks – resonates with the obstacles identified in previous research (Furinghetti & Morselli, 2011; Lee, 2019; Lesseig et al., 2019). In the Vietnamese context, these difficulties are likely exacerbated by limited practicum time and pre-service teachers' tendency to choose 'safer' topics (e.g., algebra and calculus) during teaching practice, as our participants suggested. Addressing these issues requires not only more coursework on proof pedagogy but also intentional design of practicum experiences that expose pre-service teachers to proof-intensive lessons.

Overall, the survey results clearly indicate that pre-service mathematics teachers face difficulties not only in their individual knowledge and skills but also in integrating knowledge, skills, and practical experience. Therefore, pre-service mathematics teachers need to participate in more practical activities, such as teaching (Trinh et al., 2023), classroom observation, scientific research, etc., to improve their pedagogical skills (Cuong et al., 2023). This is an important practical basis for apparently systematic training measures that focus on strengthening practice, supporting lesson design, developing assessment skills, and providing rich learning resources for pre-service mathematics teachers during their training to become mathematics teachers.

## 5. CONCLUSION

The survey results reveal that the majority of the examined pre-service mathematics teachers had a positive pedagogical belief base and relatively solid professional knowledge, but their instructional practice skills were not commensurate, especially in activity design, lesson organization, and assessment. The gap between belief and instructional practice is a problematic issue that calls for attention in current mathematics teacher training. The survey results not only reflect the current situation but also provide an important practical basis for proposing measures to develop the professional level of pre-service mathematics teachers by strengthening practical application, simulation, pedagogical feedback, and by closely linking theory with instructional practice in teacher training programs.

From a methodological perspective, this study has several limitations. First, although the questionnaire was carefully developed based on previous research and reviewed by mathematics education experts to ensure content relevance and clarity, we did not conduct a full psychometric validation of the scales, including reliability analyses or factor analyses. As a result, the findings should be interpreted with caution, and future studies are encouraged to examine the reliability and construct validity of the belief and instructional competence scales more rigorously. Second, the study relied exclusively on self-report data, which capture pre-service teachers' perceptions and self-

evaluations rather than their actual classroom practices. Observational data, lesson plans, or teaching artifacts were not collected, so the reported “instructional practices” reflect perceived competence rather than directly observed behavior.

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## REFERENCES

- Anh, H. N., & Lan, N. T. H. (2019). Một số biện pháp sư phạm nhằm phát triển năng lực dạy học cho sinh viên Sư phạm Toán Trường Đại học Tây Bắc [Some pedagogical measures aimed at developing teaching competence for pre-service mathematics teachers at Tay Bac University]. *Journal of Social Sciences*, 17, 79-88. [In Vietnamese]
- Azhar, M., & Kayani, M. M. (2017). Study of the impact of training of novice teachers in the context of transformative learning in Punjab, Pakistan. *Advanced Education*, 4(8), 84-91. <https://doi.org/10.20535/2410-8286.112533>
- Clarke, J. N. (2023). Proof of purpose. *SSRN Electronic Journal*. <https://doi.org/10.2139/ssrn.4547587>
- Cuong, T., Hung, B. D., & Phuong, B. T. H. (2023). Một số biện pháp phát triển năng lực dạy học cho sinh viên Sư phạm Toán thông qua học phần “Thực hành dạy học” [Some measures to develop teaching competence for pre-service mathematics teachers through the “Teaching practice” module]. *Vietnam Journal of Education*, 23(24), 19-24. [In Vietnamese]
- Czocher, J. A., & Weber, K. (2020). Proof as a cluster category. *Journal for Research in Mathematics Education*, 51(1), 50-74. <https://doi.org/10.5951/jresmetheduc.2019.0007>
- Er, S., & Dost, Ş. (2022). A design study to develop the proof skills of mathematics pre-service teachers. *Necatibey Eğitim Fakültesi Elektronik Fen ve Matematik Eğitimi Dergisi*, 16(1), 189-226. <https://doi.org/10.17522/balikesirnef.1081825>
- Frasier, B. J., & Panasuk, R. (2013). Proofs in mathematics education: Expectations and reality. *Journal of Curriculum and Teaching*, 2(2), 76-85. <https://doi.org/10.5430/jct.v2n2p76>
- Furinghetti, F., & Morselli, F. (2011). Beliefs and beyond: Hows and whys in the teaching of proof. *ZDM Mathematics Education*, 43(4), 587-599. <https://doi.org/10.1007/s11858-011-0316-7>
- Gower, R., Phillips, D., & Walters, S. (2009). *Teaching practice handbook*. Macmillan Education.
- Haavold, P., Roksvold, J., & Sriraman, B. (2024). Pre-service teachers’ knowledge of and beliefs about direct and indirect proofs. *Investigations in Mathematics Learning*, 16(4), 336-355. <https://doi.org/10.1080/19477503.2024.2363710>
- Hanna, G., & Knipping, C. (2020). Proof in mathematics education, 1980-2020: An overview. *Journal of Educational Research In Mathematics*, 30(S), 1-13.
- Havelková, V., Jančařík, A., & Jančaříková, K. (2022). Analysis of approaches to the use of ICT in the teaching of mathematics. *Proceedings of the European Conference on E-Learning, ECEL*, 2022-October. <https://doi.org/10.34190/ecel.21.1.739>
- Hieu, D. V. (2009). *Sử dụng một số bất đẳng thức thông dụng để chứng minh bất đẳng thức* (Luận văn thạc sĩ) [Using some common inequalities to prove inequalities (Master’s thesis)]. Faculty of Science, Thai Nguyen University. [In Vietnamese]
- Hung, P. A. (2017). Phát triển năng lực dạy học cho sinh viên đại học ngành Sư phạm Toán [Developing teaching competencies for pre-service mathematics teachers]. *Vietnam Journal of Educational Sciences*, 139, 49-52. [In Vietnamese]

- Kasanda, C. D. (1995). Instructional practice at the University of Namibia: Views from student teachers. *Zimbabwe Journal of Educational Research*, 7(1), 57-68.
- Kotelawala, U. (2016). The status of proving among US secondary mathematics teachers. *International Journal of Science and Mathematics Education*, 14(6), 1113-1131. <https://doi.org/10.1007/s10763-015-9638-1>
- Kul, U. (2018). Influences of technology integrated professional development course on mathematics teachers. *European Journal of Educational Research*, 7(2), 233-243. <https://doi.org/10.12973/eu-jer.7.2.233>
- Lee, C.-Y. (2019). Factors influencing teachers' decision making on reasoning-and-proving in Hong Kong. In U. T. Jankvist et al. (Eds.), *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education* (pp.257-264). Utrecht University and ERME. <https://hal.archives-ouvertes.fr/hal-02398450/>
- Lee, G. C.-Y. (2026). Conditions for change in pre-service teachers' beliefs about proof and proof teaching. *Mathematics Education Research Journal*, 38(1), 187-218. <https://doi.org/10.1007/s13394-025-00525-4>
- Lesseig, K., & Hine, G. (2022). Teaching mathematical proof at secondary school: An exploration of pre-service teachers' situative beliefs. *International Journal of Mathematical Education in Science and Technology*, 53(9), 2465-2481. <https://doi.org/10.1080/0020739X.2021.1895338>
- Lesseig, K., Hine, G., Na, G. S., & Boardman, K. (2019). Perceptions on proof and the teaching of proof: A comparison across pre-service secondary teachers in Australia, USA and Korea. *Mathematics Education Research Journal*, 31(4), 393-418. <https://doi.org/10.1007/s13394-019-00260-7>
- Mingus, T. T. Y., & Grassl, R. M. (1999). Pre-service teacher beliefs about proofs. *School Science and Mathematics*, 99(8), 438-444. <https://doi.org/10.1111/j.1949-8594.1999.tb17506.x>
- Noto, M. S., Priatna, N., Dahlan, J. A., & Setiyani, S. (2019). How good pre-service mathematics teacher in reading mathematical proof? *Journal of Physics: Conference Series*, 1280(4), 042032. <https://doi.org/10.1088/1742-6596/1280/4/042032>
- Son, N. L. T. (2013). Nâng cao khả năng diễn đạt và trình bày suy luận chứng minh hình học của học sinh trung học cơ sở thông qua tổ chức hoạt động trên bảng [Enhancing the ability of secondary school students to express and present geometric reasoning and proofs through whiteboard activities]. *Ho Chi Minh University of Education Journal of Science*, 54, 126-135. [In Vietnamese]
- Stylianides, G. J., Stylianides, A. J., & Moutsios-Rentzos, A. (2023). Proof and proving in school and university mathematics education research: a systematic review. *ZDM - Mathematics Education*, 56(1), 47-59. <https://doi.org/10.1007/s11858-023-01518-y>
- Tassell, J. L., Maxwell, M., Stobaugh, R., & Mittelberg, J. (2019). Math and technology leadership academy: Impact on mathematics teacher sense of efficacy. *International Journal of Innovation in Science and Mathematics Education*, 27(3), 1-13. <https://doi.org/10.30722/ijisme.27.03.001>
- Tu, V. C., & Chi, N. Y. (2024). Một số nghiên cứu về hoạt động phát triển chuyên môn cho giáo viên môn khoa học tự nhiên [Several studies on professional development activities for natural science teachers]. *Vietnam Journal of Education*, 24(7), 30-36. [In Vietnamese]
- Thang, N. C. (2011). Một số phương thức rèn luyện cho sinh viên sư phạm toán kỹ năng biến đổi thông tin khi dạy học các môn Toán sơ cấp [Some methods for pre-service mathematics teachers in information transformation skills when teaching elementary mathematics subjects]. *Ho Chi Minh University of Education Journal of Science*, 27, 68-77. [In Vietnamese]
- Thanh, H. T. (2019). Rèn luyện các hoạt động trí tuệ chung cho học sinh thông qua tìm lời giải và khai thác bài toán chứng minh hình học trung học cơ sở [Develop general intellectual skills in students through finding solutions and exploring proof-based geometry problems at the middle school level]. *TBU Journal of Science*, 8, 1-8. [In Vietnamese]
- Trình, Đ. T., Cuong, T. V., & Tai, H. V. (2023). Một số biện pháp phát triển năng lực dạy học tích hợp cho sinh viên ngành sư phạm Toán [Some measures to develop integrated teaching competencies for pre-service mathematics teachers]. *Vietnam Journal of Education*, 23(24), 9-13. [In Vietnamese]